# Multi Objective Optimization Course
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Europska unija
"Zajedno do fondova EU"
EUROPSKI STRUKTURNI I INVESTICIJSKI FONDOVI
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# outline

- Introduction to combinatorial optimization   30min
- Heuristic and Evolutionary Algorithms 30min
- Multiple Objective Optimization 1h

- Lab : Planning a ROV mission 2h

# Introduction to discrete optimization

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# Discrete optimization

- **Many choices**
  - Not obvious … many aspects to take into consideration …

Problem : How to find the best **solution** according to some **criteria** ?

- You can enumerate all of them (discrete problem)

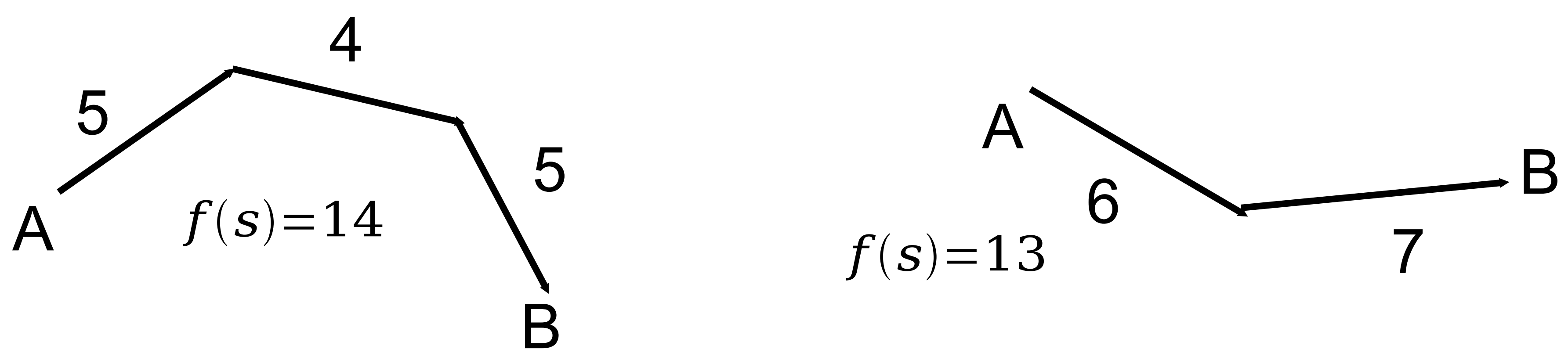
# Combinatorial optimization introduction

- Optimization problem : *find the best solution to a given problem among a set of feasible ones, according to some optimization criteria*
  - *S :* solution space
  - *f(S) :* function (objective function) for evaluation solution quality – can be maximized or miinimized

Oops, a single place !

# Combinatorial optimization examples

- Path finding : *shortest path within a graph between couples of nodes*
  - *S :* all possible paths
  - *f(S) : path length* (to minimize !)




- Graph optimization

# Combinatorial optimization examples

- Travelling salesman problem : *visit every town a single time and come back to the starting point*
  - *S :* all possible roundtrips (tours)
  - *f(S) : roundtrip length* (to minimize !)

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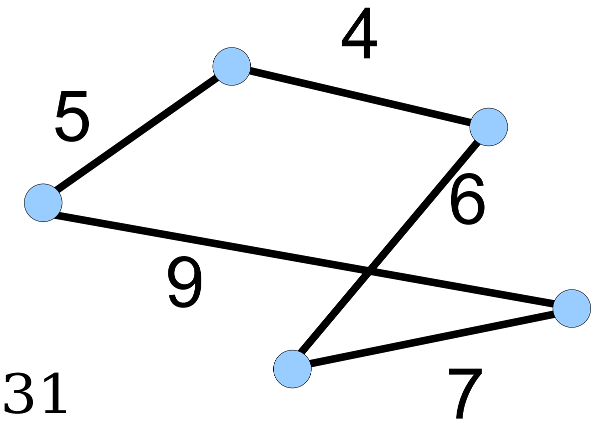
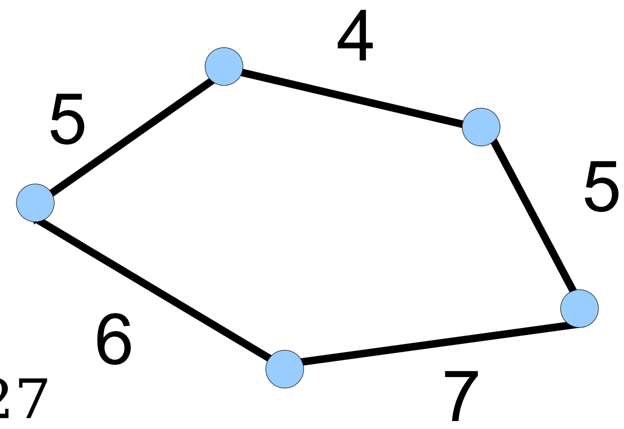



$f(s)=31$




$f(s)=27$

# Combinatorial optimization examples

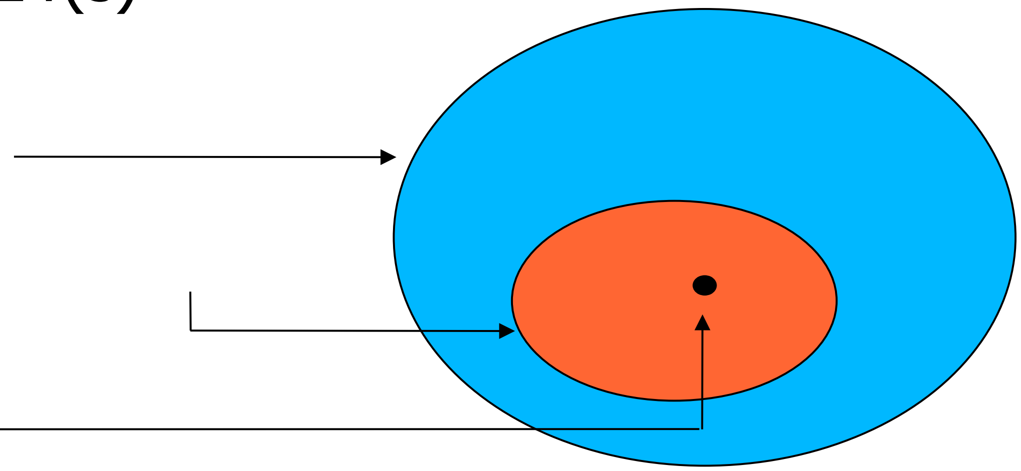
- A carpenter can make at most 6 seats and 3 tables by day (8 hours of work)
  - He sells a table \$90 (working 1h15)
  - A seat, \$50 (working 45mn)
- How to maximize his benefit ?

$$
\begin{cases}
90t + 50c = f(s) \\
75t + 45c \leq 480 \\
0 \leq t \leq 3 \\
0 \leq c \leq 6
\end{cases}
$$

- *Linear programming :* simplex method with $O(2^n)$ complexity

# Combinatorial optimization framework

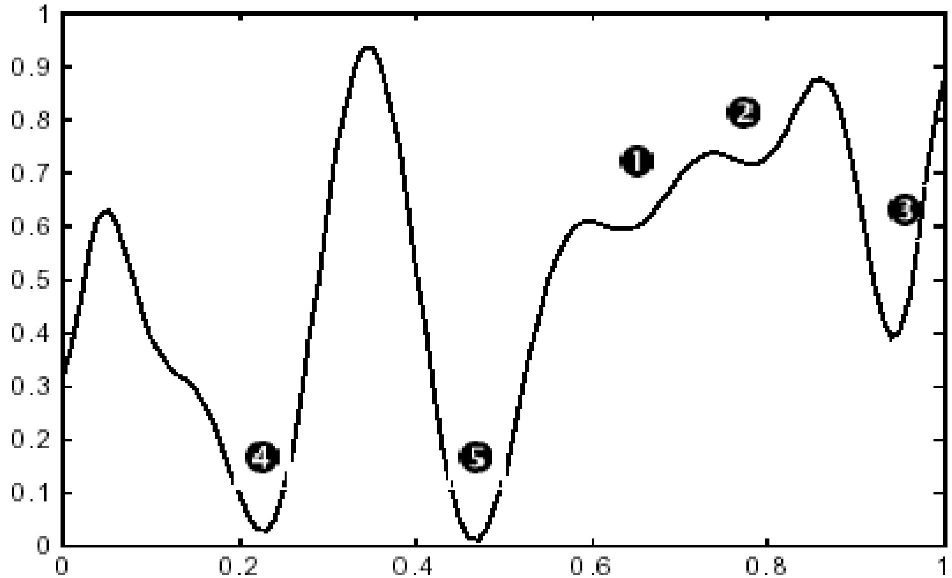
- Solution space $S \subseteq X$
- Objective function (e.g. min) $f : X \rightarrow \mathbb{R}$
- Find $s^* \in S$ s.t. $\forall s \in S \;\; f(s^*) \leq f(s)$
- X , solution space
- S , **feasible** solution space
- $s^*$ , optimal solution



# Combinatorial optimization
# local sub optimal solutions

- Can fall into a local minimum as 1, 2, 3, 4, 5 (5 is the best :-)
- Must explore the whole solution space
- Not only neighbourhood
- Example : minimize a continous function on a single variable

f(x)




Minimize f(x) : $X \to [0..1]$

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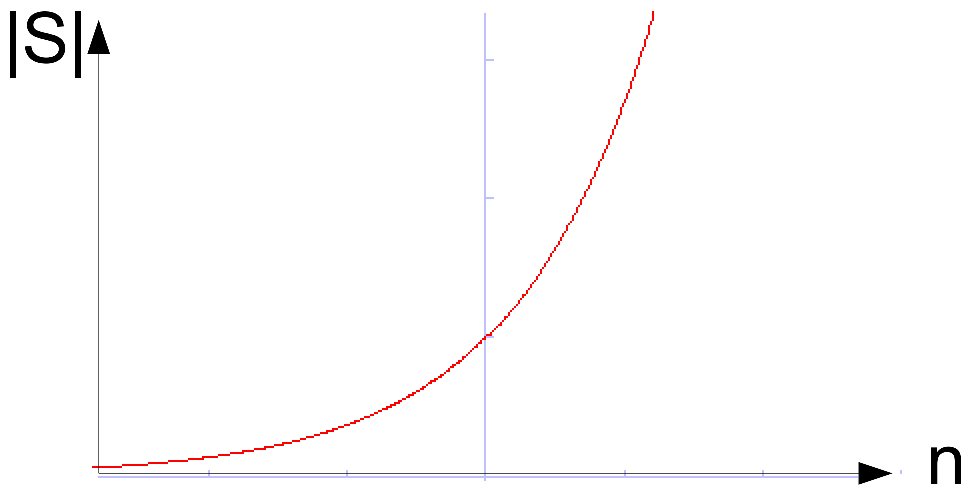
# Combinatorial optimization combinatorial explosion

- Problem of the size of S related to the size of data
  - TSP (n-1)! / 2
  - Bi partitioning $2^n$
  - 0-1 Integer Programming $2^n$




- S size is exponential

# Combinatorial optimization
# combinatorial explosion

- Enumerate all solutions : often impossible

| *Complexity* | *N = 1* | *N = 10* | *N = 100* | *N = 1000* | *N = 10000* |
|---|---|---|---|---|---|
| *log N* | 0 ms | 1 ms | 2 ms | 3 ms | 4 ms |
| *N* | 1 ms | 10 ms | 0.1 s | 1 s | 10 s |
| $N^2$ | 1 ms | 0.1 s | 10 s | 17 min | 28 hours |
| $N^3$ | 1 ms | 1 s | 17 min | 12 days | 32 years |
| $e^N$ | 3 ms | 22 s | 9 $10^{32}$ years ! | Long time | Very long time |

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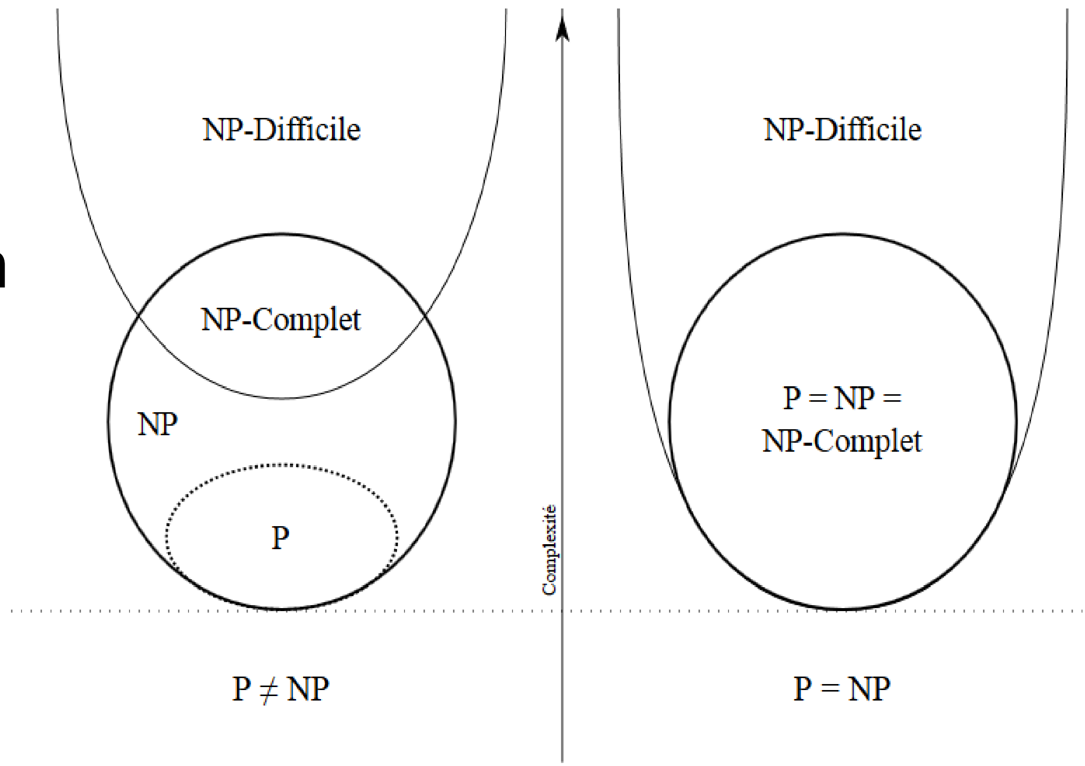
*If 1000 solutions evaluated per sec*

# Classes of problems
# Complexity

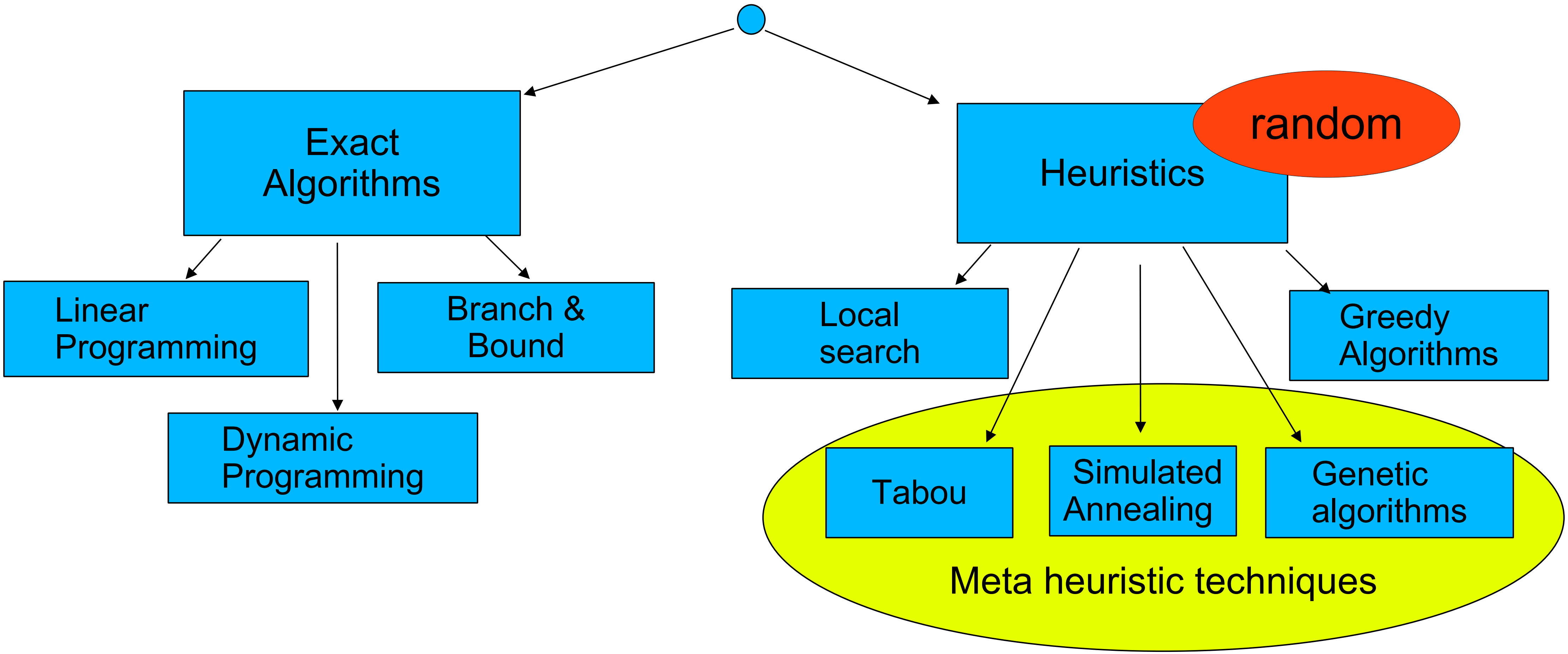
**decision** problem p :

- p ∈ P class: polynomial algorithms for solving p in polynomial time
- p ∈ NP class: no known polynomial algorithm, but checking of solution in polynomial time. [ Maybe P = NP ]
- iff all problems in NP can be reduced p by a polynomial transformation
  - If p ∈ NP, p ∈ NP-complete class (⟶ hardest problems in NP)
  - If p ∉ NP, p ∈ NP-hard class

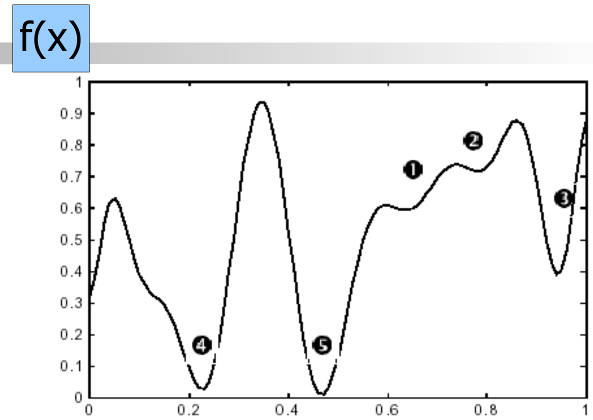



- LP ∈ P
- MILP ∈ NP-hard

# Combinatorial Optimisation
# search techniques



# Combinatorial Optimisation
# local vs. global techniques

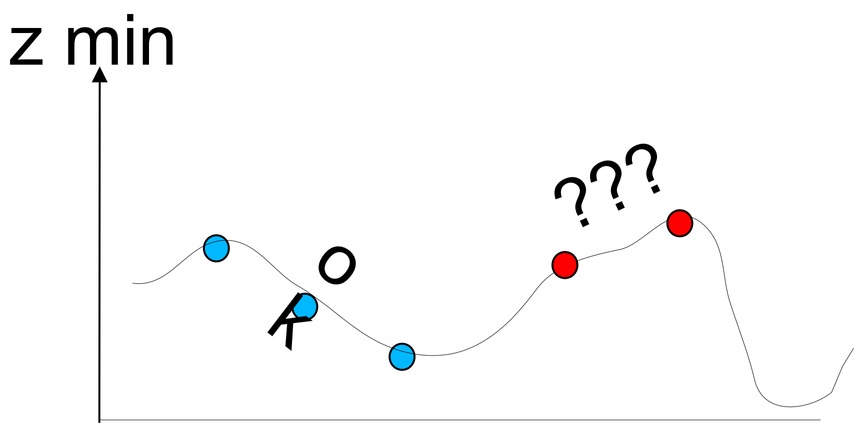



Minimize f(x) : X $\rightarrow [0..1]$

- Remember local minimum problem
- Choice between
  - Improve current solution
  - Exploring the whole search space

Exploitation ⟷ Exploration

- Trade off politics design



# Combinatorial Optimisation
# exact vs. approximative techniques

- Practically speaking :
  - *Don't always need* the best solution
  - *but* have a *good* solution and eventually a guarantee on the quality loss

- If exact solution, exact method (sometimes impractical or too much time consuming)
- If appproximation
  - Heuristics (allowing discovery based on random mechanism)
  - Meta-heuristics (Frameworks for derivating specialized heuristics)

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# Large scale problems
# Heuristics useful

- Approximative result, but
  - Sometimes only available method (*e.g* program optimization)
  - Or exact methods for approximative model only (*e.g* circuit testing)
- Usefullness
  - Combinatorial explosion
  - Multiple or fuzzy objectives
  - Variability (robustness)
  - Fast runtimes more important than performance

# Large scale problems
# Parallelism

- Too large problems or need for faster runtimes
- Availability of parallel computers (multicore, NOW)

- Possible parallelization
  - Search space partitioning : positive or negative anomalies favorables ou défavorables depending on search strategy and fitness function
  - Centralized or distributed implementation
  - Z update problem