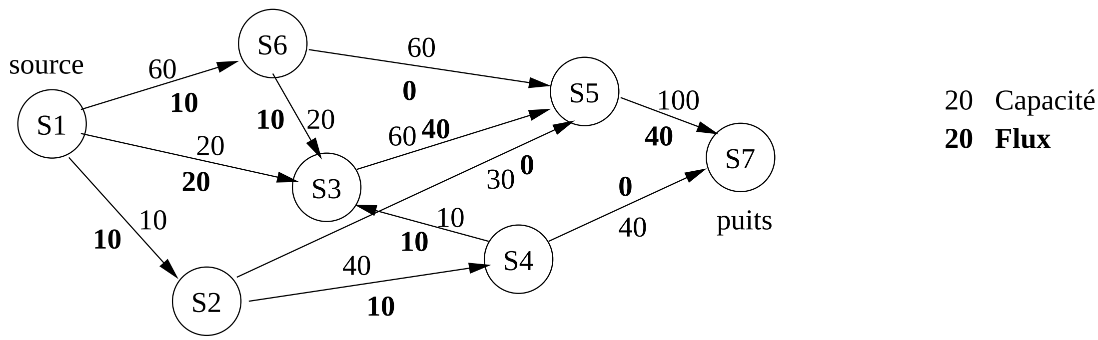
Exercices
Graphs

Focus : Floyd algorithm, network flows

1 Network flow

Q1: What are the properties of flows ?



Q2: Compute the residual network of the flow network below.

Q3: Is it finished ? Justify.

Q4: If not finished, do it.

2 Maximum flow of minimal cost

Each edge can be associated to a capacity, but also to a cost $p(u)$ (cost per unit of flow carried by the edge u within a given solution). The max flow cost is then $\sum_{u \in E} f(u).p(u)$.

Busacker and Gowen algorithm : Instead of weighting the edges by capacities in the residual network, these are associated to costs (negative costs on backward edges). And the augmenting path is a shortest path instead of a max capacity path. This technique allows to solve the max flow min cost problem.

Application to the transport problem defined as follows : One wants to carry a max of stuff at min cost from a set of warehouses or factories to a set of retail locations at minimal cost. Each warehouse stores an amount of stuff, each retail point correspond to an amount of demand. A network of transportation with limited capacities on edges connects the differents places.

Q5: Modelize le problem described below with a flow network.

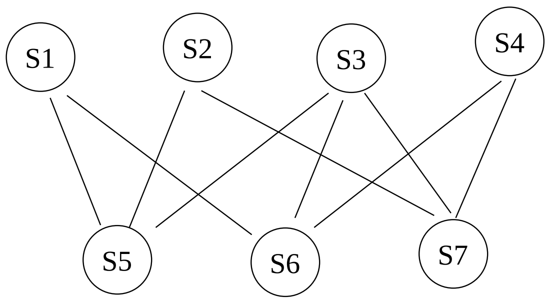
One wants to send oysters from Oléron (7 tons) and Cancale (12 tons) to Rungis and Lyon (resp. 9 and 15 tons needed). From Oléron, you can go to Lyon or Rennes. From Cancale, to Rennes, Rungis or Lyon. Trucks allow to send at most 5 t from Oléron towards Rennes, 14 from Oléron to Lyon, 7 from Cancale to Rennes, 4 from Cancale to Rungis, 2 from Cancale to Lyon, and 6 from Rennes towards Rungis. Last, transport cost rates (per ton) are respectively of 100, *400, 200, 200, 700 and 0 dollars for mentioned journeys.*

Q6: Transform this graph into a flow network. Which capacity and cost is to be set for added edges ?

Q7: Solve the problem with Busaker and Gowen algorithm.

3 Matching

The goal is to solve the maximal cardinality matching problem in a bipartite graph. It consists in finding a maximum number of coules (A, B) of nodes belonging to the 2 parts of the graph, s. t. A and B are connected by an edge (compatibility edge).



Q8: How to modelize it as a max flow problem ? solve it for the given graph.

Q9: A congress must gather research teams from different laboratories. Each lab i send to the conference n_i representatives. The social event (dinner) requires d tables, each table j is for d_j attendees. 2 people from the same lab can not be placed at the same table in order to maximaxe the exchanges during the dinner.
Modelize the problem in general case. Is there a solution for 4 labs (4, 5, 3, 6 researchers resp.), and 5 tables (3, 5, 2, 6 and 4 attendees resp.).

4 Simple Time Problem

The floyd algorithm allows to exhibit cycles of negative total length within a graph. According to constraints below, can we set a date for each of the events ? (recall : oriented edge $(i \rightarrow j)$ for constraint $x_j - x_i \leq a$)

$$10 \leq X_1 \leq 60 \tag{1}$$
$$15 \leq X_2 \leq 25 \tag{2}$$
$$X_1 - X_2 \leq 20 \tag{3}$$
$$X_3 \geq 20 \tag{4}$$
$$X_1 - X_3 \geq 30 \tag{5}$$

Q10: Draw the STP graph (include node X_0 for date 0) representing the set of constraints above.

Q11: What are the results of the algorithm. What does it mean ?